A Partial-Order Based Model to Estimate Individual Preferences using Panel Data

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Joint with:

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NYU Stern School of Business  
School of Business, Torquato di Tella University, Argentina

Dmitry Mitrofanov  
NYU Stern School of Business
Effective operational decisions require accurate demand predictions.

**Goal:** predict customer’s choice from a menu of items

**Existing methods in operations**

**Data**
- sales transactions
- product availability

**Models**
- choice models: accounting for substitution

**Limitations**
- ignores
  - repeated customer visits
  - *considered* items

Substitution rates:
- 15 - 45% in airlines [Ja et al. '01]
- 45% in retail [Gruen et al. '02]
Our focus: personalized demand predictions using panel data

Panel data sales transactions tagged by customer id

Challenges

- very few observations per customer
- product unavailability [stock-outs]
- switching [price/quantity promotion]
- latent consideration sets
Our focus: personalized demand predictions using panel data

Panel data  sales transactions tagged by customer id

<table>
<thead>
<tr>
<th>Challenges</th>
<th>Our approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>• very few observations per customer</td>
<td>use partial orders to efficiently extract preference info</td>
</tr>
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Our focus: personalized demand predictions using panel data

| Panel data | sales transactions tagged by customer id |

**Challenges**

- very few observations per customer
- **product unavailability** [stock-outs]
- switching [price/quantity promotion]
- latent consideration sets

**Our approach**

choice model
both utility- and rank-based
Our focus: personalized demand predictions using panel data

Panel data sales transactions tagged by customer id

Challenges

- very few observations per customer
- product unavailability [stock-outs]
- switching [price/quantity promotion]
- latent consideration sets

Our approach

behavioral model
Our focus: personalized demand predictions using panel data

Panel data: sales transactions tagged by customer id

Challenges:
- very few observations per customer
- product unavailability [stock-outs]
- switching [price/quantity promotion]
- latent consideration sets

Contributions:
- methodology for accurate predictions
- performance gains on IRI dataset
Application to personalized promotions

Personalized promotions:
- drive up sales
- increase both visits & basket size
- reduce competition
- stronger cust. relationship
- price discrimination
Related work: Rich work spanning various areas

**Operations:**

*Estimation:* [Farias, Jagabathula, Shah ’12], [Haensel, Koole ’11], [van Ryzin, Vulcano ’13], [Jagabathula, Rusmevichientong ’13], …

*Optimization:* [Mahajan, van Ryzin ’01], [Farias, Jagabathula, Shah ’13], [Papers by Rusmevichientong, Topaloglu, and Gallego], …

*Airline RM:* [Zhang, Cooper ’06], [Chaneton, Vulcano ’11], [Kunnumkal ’12], …

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**Marketing:**

[Allenby, Lenk ’95], [Chib, Seetharaman, Strijnev ’04], [Chintagunta ’92], [Erdem ’93], [Erdem, Imai, Keane ’03], [Guadagni, Little ’83], [Hendel, Nevo ’06], [Jedidi, Mela, Gupta ’99], [Lattin ’87], …

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**Machine Learning/Statistics:**

[Lu, Boutilier ’11], [Lebanon, Mao ’08], [Guiver, Snelson ’09], [Jagabathula, Shah ’08], [Jagabathula, Shah ’11], [Meila, Chen ’10], [Meila, et.al. ’07], [Kondor, Barbosa ’10], [Huang, Guestrin ’10], …

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Focus is on:

- decision-making
- estimation from aggregated transactions

Our focus: panel data & individual predictions

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How covariates affect choice

We complement by enriching inferences through partial orders

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Methodological connections

We borrow and contribute
A Partial-Order Based Model to Estimate Individual Preferences Using Panel Data

1. Model
   assumptions, data for estimation

2. Modular inference framework
   consideration set, clustering of DAGs, ML estimation

3. Numerical results
   results on real-world panel data from IRI dataset

4. Summary/Conclusions
   takeaway messages
1. Model
   assumptions, data for estimation

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Panel data: sales transactions tagged with customer ids

$n$ products | $m$ customers
---|---
mutually substitutable | make repeated purchases
0 is no-purchase | 

Observations: for each customer and time period

<table>
<thead>
<tr>
<th>Offer set</th>
<th>Purchased product</th>
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<td>Purchase $i_t$</td>
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<td>$t = 1$</td>
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<tr>
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<tr>
<td>$t = 3$</td>
<td>1 2 3 4 5 6 7</td>
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</table>
IRI Academic Data Set

Weekly store sales and consumer panel data

31 product categories
11 years
2001 - 2011

We analyzed year 2007 panel data

29 product categories
84K user-category combinations
1.2M transactions across 52 weeks
<table>
<thead>
<tr>
<th>Category name</th>
<th># Vendors</th>
<th># Customers retained</th>
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<td>Beer</td>
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**Training data summary**

- aggregate items by vendors
- training: first 26 weeks
- retain customers ≥ 2 sales
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**Training data summary**
- aggregate items by vendors
- training: first 26 weeks
- retain customers $\geq 2$ sales

**After pre-processing**
- **64K** user-category combinations
- **1.1M** transactions across 52 weeks
- **36** vendors
- **2.2K** customers

Average across 29 categories
Choice model: preferences consistent across purchase instances

Customers characterized by partial-order (DAG)
distribution over rankings

- samples rank list consistent with DAG
- purchases most preferred offered item

 Directed Acyclic Graph (DAG) bag of comparisons
Choice model: preferences consistent across purchase instances

Customers characterized by partial-order (DAG)

- samples rank list consistent with DAG
- purchases most preferred offered item

Subsumes existing methods with empty partial order

Offer set: \{1, 3, 4\}
Purchase: 3 or 4
choice depends on sampled rank list

0 \rightarrow 2 \rightarrow 3
0 \rightarrow 2 \rightarrow 0
0 \rightarrow 2 \rightarrow 4
0 \rightarrow 4 \rightarrow 1
0 \rightarrow 4 \rightarrow 5
0 \rightarrow 2 \rightarrow 1
0 \rightarrow 2 \rightarrow 5

2
3
4
1
5
Model benefits: efficiently extracts preference information from data

Infer preference relations with fewer samples

RUM model
- Rank lists differ across periods
- prefers 1 or 2 each period
- many samples needed to estimate

Strong preferences
- Rank lists consistent with partial order
- prefers 1 over 2 every period
- fewer samples needed to estimate

- Rank list same across periods
- only few samples needed to estimate
Model benefits: efficiently extracts preference information from data

Infer preference relations with fewer samples

Heterogeneity through DAGs yields parsimonious models

Two customer types:

- **type 1**
  - empty DAG
  - random choice

- **type 2**
  - random choice from 1, 2, and 5

For classical DAGs:
- Need two MNL

For DAGs:
- One MNL and two DAGs
Model benefits: efficiently extracts preference information from data

Infer preference relations with fewer samples

Heterogeneity through DAGs yields parsimonious models

Enrich revealed comparisons with inferred comparisons

<table>
<thead>
<tr>
<th></th>
<th>purchase</th>
<th>offer set</th>
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<tbody>
<tr>
<td>( t = 1 )</td>
<td>4</td>
<td>{2, 4, 7}</td>
</tr>
<tr>
<td>( t = 2 )</td>
<td>2</td>
<td>{1, 2, 3, 5}</td>
</tr>
</tbody>
</table>

transitivity allows inferring edges not “directly revealed”

enriched preference data  more accurate predictions
Consideration set: customers may not consider all the products on offer

Customers may choose from smaller consideration set

Avoids inferring spurious preference relations
Model: Customers sample rank lists from a distribution

Market with $K$ segments
- type $k$ samples DAG from distribution $f_k$
- uses $f_k$ to sample rank lists consistent with DAG

Sales over $T$ periods
- product universe (almost) constant
- customer population (almost) constant

Objective
- predict purchase of each customer
1. Model
   assumptions, data for estimation

2. Modular inference framework
   consideration set, clustering of DAGs, ML estimation

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4. Summary/Conclusions
   takeaway messages
Three-step modular framework for inference

**Step 1** Construct customer DAGs

**Step 2** Cluster the DAGs

**Step 3** Fit choice models to clusters

Steps do not depend on assumptions of previous steps
Three-step modular framework for inference

- **Step 1**: Construct customer DAGs
  - Construct the DAGs

- **Step 2**: Cluster the DAGs
  - Cluster the DAGs

- **Step 3**: Fit choice models to clusters
  - Fit choice models

Steps do not depend on assumptions of previous steps

- Technique: clustering technique
- Model for consideration set: Approx ML estimation
Standard assumption: consider everything on offer

$t = 1; offer set = \{1, 2, 3\}$

consideration set

purchased

$t = 2; offer set = \{2, 4, 7\}$

consideration set

purchased

26
Inertial assumption: effect of stock-outs, promotions, and brand inertia captured

Standard assumption: potentially unrealistic for frequently purchased products

Behavioral principle: customers do not consider unless they have to
captures brand choice inertia (short-term loyalty)
[Jeuland '79]

Behavioral rules

to infer preferences among full-priced items
prev purchase
stocked-out
in-stock
consideration set
offer set
prev purchase + promoted items
Inertial assumption: example

$t = 1; \text{ offer set } = \{1, 2, 3\}$

$t = 2; \text{ offer set } = \{2, 4, 7\}$
Inertial assumption: example (contd.)

\[ t = 1; \text{offer set} = \{1, 2, 3\} \]

consideration set

\[
\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\end{array}
\]

purchased

\[ t = 2; \text{offer set} = \{2, 4, 7\} \]

consideration set

\[
\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\end{array}
\]

purchased
Inertial assumption: example (contd.)

$t = 2; offer set = \{2, 4, 7\}$

consideration set

\[ \begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\end{array} \]

purchased

$t = 3; offer set = \{2, 3, 4, 5, 6\}; promoted items = \{3, 5, 6\}$

consideration set

\[ \begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\end{array} \]

purchased
Censored assumption: allow deviations from the inertial assumption

- purchase unexplained by inertial assumption → consideration set unobserved/censored

RULE: update “prev purchase” (sticky product); don’t add edges to DAG
Censored assumption: example

$t = 1; \text{offer set} = \{1, 2, 3\}$

consideration set

1 2 3

purchased

4 5 6 7

DAG does not change

$t = 2; \text{offer set} = \{1, 2, 4\}$

consideration set

1 2 3

purchased

4 5 6 7

DAG does not change
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Avg. number of customers with non-empty DAGs
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<tr>
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<td><strong>Avg. number of customers</strong></td>
<td><strong>31%</strong> standard <strong>39%</strong> inertial <strong>71%</strong> censored</td>
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<tr>
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<tr>
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<tr>
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<tr>
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<tr>
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<td>Frozen Pizza</td>
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<td>Margarine/Butter</td>
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</table>

Avg. number of edges in the DAGs

- Standard: 19
- Inertial: 11
- Censored: 17
Three-step modular framework for inference

Step 1: Construct customer DAGs

Step 2: Cluster the DAGs

Step 3: Fit choice models to clusters

Steps do not depend on assumptions of previous steps
Clustering of user DAGs:
partial orders in same cluster are “close” to central ranking

Each customer represented by a DAG

Clustering assumptions
- Each customer mapped to a single cluster
- Every cluster represented by a central ranking
- Customer DAGs in same cluster “close” to central ranking

Distance between DAG of customer central ranking
(defined over resp. transitive closures)

\[ d(c, \sigma) = (\# \text{ disagreements}) - (\# \text{ agreements}) \]
\[ = 2 (\# \text{ disagreements}) - (\# \text{ edges in DAG of } c) \]
MIP to cluster DAGs and find centroid ranking of each cluster

\[
\begin{align*}
\text{min} & \quad \sum_{c} \sum_{(i,j) \in c} (2w_{ijc} - m_c) \\
\text{s.t.} & \quad \sum_{k=1}^{K} T_{ck} = 1, \\
\text{counting disagreements} & \quad w_{ijc} \geq \delta_{ijk} + T_{ck} - 1, \forall (i, j) \in c, \forall k, \\
\text{total order between } i \text{ and } j & \quad \delta_{ijk} + \delta_{jik} = 1, \forall i, j, k, \\
\text{transitivity constraint} & \quad \delta_{ijk} \geq \delta_{irk} + \delta_{rjk} - 1, \forall i, j, k, \\
0 & \leq w_{ijc} \leq 1, \forall i, j, c, \\
\delta_{ijk} & \in \{0, 1\}, \\
T_{ck} & \in \{0, 1\}, \forall c, k.
\end{align*}
\]

indicator that edge \((i,j)\) in \(c\) is a disagreement with centroid in an allocation

indicator that \(c\) is assigned to cluster \(k\)

bounds and integrality constraints
Three-step modular framework for inference

Step 1: Construct customer DAGs
- Construct DAGs

Step 2: Cluster the DAGs
- Cluster DAGs

Step 3: Fit choice models to clusters
- Fit choice models

Steps do not depend on assumptions of previous steps

Techniques:
- Approximate ML estimation
MNL model has closed-form expression for forest of trees with one root

Likelihood of DAG

\[
\prod_{i=1}^{n} \frac{v_i}{v_i + \sum_{j \in T_i} v_j}
\]

prob of tree = (prob of subtree 2) x (prob of subtree 3) x \( \frac{v_1}{v_1 + \cdots + v_{11}} \)

prob of subtree 2 = (prob of subtree 4) x (prob of subtree 5) x \( \frac{v_2}{v_2 + v_4 + v_5 + v_8 + v_9} \)

\vdots
A Partial-Order Based Model to Estimate Individual Preferences Using Panel Data

1. Model
   assumptions, data for estimation

2. Modular inference framework
   consideration set, clustering of DAGs, ML estimation

3. Numerical results
   results on real-world panel data from IRI dataset

4. Summary/Conclusions
   takeaway messages
Benchmark models compared: LC-MNL and RPL

**k-latent class MNL (LC-MNL)**

- Sample class membership; follow MNL for that class
- # of parameters = $k \times n$
- EM-based regularized max likelihood estimation
- Best performance up to $k = 10$ reported

**Random Parameters Logit (RPL)**

- Sample MNL parameters $\sim$ multivariate normal
- # of parameters = $2 \times n$
- Max simulated likelihood estimation (MSLE)
- Computationally intensive
Experiments conducted: one step-ahead prediction

\[ U = \text{user set} \quad N = \text{product set} \quad T = \text{# of discrete time periods} \]

For any \( t = 1, 2, 3, \ldots, T \)

**Given**
- everything until time period \( t \)
  - (offer sets, promoted items, purchases of users in \( U \))
- \( S_{t+1} = \text{offer set in period } t+1 \)
- \( P_{t+1} = \text{promoted items in period } t+1 \)
- \( U_{t+1} = \text{users purchasing in period } t+1 \)

**Prediction**
- \( f_u(i, t+1) = 1 \) if \( i \) has highest choice probability for \( u \) in period \( t+1 \)
- for all \( u \in U_{t+1}, i \in S_{t+1} \)

**X2 score**
\[
\chi^2 \text{ score} = \frac{1}{|U||N|} \sum_{u \in U, i \in N} \frac{(n_{ui} - \hat{n}_{ui})^2}{0.5 + \hat{n}_{ui}}, \quad \hat{n}_{ui} = \sum_{t \in T} f_u(i, t)
\]

**miss rate**
\[
\text{miss rate} = \frac{1}{|U||T|} \sum_{u \in U, t \in T} I[f_u(a_{j_u, t}) \neq 1]
\]

Similar to \( \chi^2 \text{ score} \) \( \frac{(O - E)^2}{E} \)

lower is better
improvement attributed to DAGs

• benchmarks have more parameters
• time: our method ~ 10 secs, RPL ~ 67 minutes
Lower is better

Boost due to
- accounting for behavioral effects
- clustering of DAGs
Improvements higher for categories with higher “brand loyalty scores”

Loyalty score = fraction of purchases from most frequently purchased brand
Results are similar for miss rate metric
Results are similar for miss rate metric
Improvements over Gudagni-Little (GL) model for inertial and censored

Improvements obtained even after accounting for state
Improvements over Gudagni-Little (GL) model for standard and censored

GL performs well for inertial because their “loyalty” consistent with GL assumptions
1. Model
   assumptions, data for estimation

2. Modular inference framework
   consideration set, clustering of DAGs, ML estimation

3. Numerical results
   results on real-world panel data from IRI dataset

4. Summary/Conclusions
   takeaway messages
## Summary and key findings

### Key Contribution

Methodology to estimate individual preferences from panel data through DAGs and choice inertia.

### Main Takeaways

<table>
<thead>
<tr>
<th>DAGs and behavioral effects</th>
<th>DAGs and behavioral effects significantly boost accuracy</th>
</tr>
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<tbody>
<tr>
<td>PO+Censored</td>
<td>best for X2 metric, comparable/best for miss-rates</td>
</tr>
<tr>
<td></td>
<td>coverage: 71% of customer DAGs</td>
</tr>
<tr>
<td>PO+Standard</td>
<td>best for miss-rates</td>
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