On High Spatial Reuse Link Scheduling in STDMA Wireless Ad Hoc Networks

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***Abstract*—We consider the point-to-point link scheduling problem in Spatial Time Division Multiple Access (STDMA) wireless ad hoc networks, motivate the use of spatial reuse as performance metric and provide an explicit characterization of spatial reuse. We assume uniform transmission power at all nodes and propose an algorithm based on a graph model of the network as well as Signal to Interference and Noise Ratio (SINR) computations. Our algorithm achieves higher spatial reuse than existing algorithms, without compromising on computational complexity.**

I. Introduction

The key to increasing the capacity of wireless ad hoc networks is to allow concurrent communication between entities which are reasonably far from each other. A prevalent scheme for channel reuse is Spatial Time Division Multiple Access (STDMA) [1], in which time is divided into fixed-length slots that are organized cyclically. An STDMA schedule describes the transmission rights for each time slot in such a way that communicating entities assigned to the same slot do not collide. STDMA scheduling algorithms can be classified into link scheduling and broadcast scheduling algorithms [2]. This paper focuses on centralized Signal to Interference and Noise Ratio (SINR) based *link* scheduling for STDMA wireless ad hoc networks, while SINR based broadcast scheduling for STDMA wireless ad hoc networks has been considered in [3].

Several innovative algorithms, both centralized [2], [4], [5], [6] and distributed [7], have been proposed in the literature for generating minimum length STDMA link schedules. Existing work on centralized STDMA link scheduling algorithms can be broadly classified into two categories: graph-based scheduling algorithms and SINR-based scheduling algorithms. Graph-based scheduling algorithms [2], [4] assume a limited knowledge of the interference and result in low throughput, while SINR-based scheduling algorithms [5], [6] require a complete knowledge of the interference and lead to higher throughput. However, all these algorithms only consider *schedule length* as their performance metric.

In this paper, we adopt a different performance metric for STDMA link scheduling: *spatial reuse*, which is defined as the average number of successful transmissions per time slot. We argue that spatial reuse is directly proportional to network throughput. On the other hand, there is no known relation between schedule length and network throughput. Consequently, algorithms which minimize schedule length [2], [5], [6] do not necessarily maximize network throughput. The notion of spatial reuse has been addressed previously, though in different contexts [8], [9]. Unlike [8], [9], we motivate the importance of spatial reuse in link scheduling in STDMA wireless networks and provide an explicit characterization of spatial reuse.

This research was supported by Department of Science and Technology (Project Code 05DS025) and Bharti Centre for Communication, IIT Bombay.

Our system model, an STDMA wireless ad hoc network with uniform transmission power at all nodes, is different from those considered in existing work on SINR-based scheduling [5], [6]. For example, in [5], the authors only consider wireless networks using link-layer reliability protocols, i.e., a packet transmission is defined to be successful if and only if both data and acknowledgment packets are received successfully. Though the authors of [6] allow non-uniform transmission power at all nodes and develop a novel algorithm, their algorithm is impractical because it does not take into account hardware-imposed constraints on maximum transmission power. Our assumption of uniform transmission power in a wireless ad hoc network is motivated by [10], wherein the authors show that high data rates are achievable in a wireless ad hoc network by allowing a node to transmit to only one neighboring node at fixed peak power in any time slot.

Our primary contribution is to propose a link scheduling algorithm for an STDMA wireless ad hoc network with uniform transmission power at all nodes. The proposed algorithm is based on a communication graph model of the network as well as SINR computations. To the best of our knowledge, there is no existing work on SINR-based link scheduling for a general STDMA wireless ad hoc network with constrained transmission power. Hence, we compare the performance of our algorithm with graph-based algorithms only. We show that the proposed algorithm has high spatial reuse and low computational complexity compared to existing algorithms.

The rest of the paper is organized as follows. In Section II, we describe our system model along with the physical and protocol interference models, discuss the limitations of graph-based scheduling algorithms and formulate the problem. Section III describes the proposed SINR-based link scheduling algorithm. The performance of our algorithm is evaluated in Section IV and its computational complexity is derived in Section V. We conclude in Section VI.

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II. System Model

Consider an STDMA wireless ad hoc network with N static nodes (wireless routers) in a two-dimensional plane. During a time slot, a node can either transmit, receive or remain idle. We assume homogeneous and backlogged nodes. Let:

$$(x_j, y_j) = \text{Cartesian coordinates of } j^{th} \text{ node} =: \mathbf{r}_j$$
$$P = \text{transmission power of every node}$$
$$N_0 = \text{thermal noise power spectral density}$$
$$D(j,k) = \text{Euclidean distance between nodes } j \text{ and } k$$

The received signal power at a distance D from the transmitter is given by $\frac{P}{D^\alpha}$, where α is the path loss factor. A link schedule assigns sets of links to time slots. Specifically, a link schedule for the STDMA network is denoted by $\Psi(C, \mathcal{S}_1, \cdots, \mathcal{S}_C)$, where

$$C = \text{number of slots in the link schedule}$$
$$\mathcal{S}_i = \text{set of transmitter-receiver pairs which can communicate concurrently in the } i^{th} \text{ slot}$$
$$:= \{t_{i,1} \to r_{i,1}, \cdots, t_{i,M_i} \to r_{i,M_i}\}$$

where $t_{i,j} \to r_{i,j}$ denotes a packet transmission from node $t_{i,j}$ to node $r_{i,j}$ in the i^{th} slot. Note that $t_{i,j}, r_{i,j} \in \{1, \ldots, N\}$ and $M_i = |\mathcal{S}_i|$. The SINR at receiver $r_{i,j}$ is given by

$$\text{SINR}_{r_{i,j}} = \frac{\frac{P}{D^\alpha(t_{i,j}, r_{i,j})}}{N_0 + \sum_{\substack{k=1 \\ k \neq j}}^{M_i} \frac{P}{D^\alpha(t_{i,k}, r_{i,j})}} \quad (1)$$

We define the Signal to Noise Ratio (SNR) at receiver $r_{i,j}$ by

$$\text{SNR}_{r_{i,j}} = \frac{P}{N_0 D^\alpha(t_{i,j}, r_{i,j})} \quad (2)$$

A. Physical and Protocol Interference Models

According to the *physical interference model* [11], $t_{i,j} \to r_{i,j}$ is successful if and only if (iff) the SINR at receiver $r_{i,j}$ is greater than or equal to a certain threshold γ_c, termed as communication threshold.

$$\frac{\frac{P}{D^\alpha(t_{i,j}, r_{i,j})}}{N_0 + \sum_{\substack{k=1 \\ k \neq j}}^{M_i} \frac{P}{D^\alpha(t_{i,k}, r_{i,j})}} \geqslant \gamma_c \quad (3)$$

According to the *protocol interference model* [11], $t_{i,j} \to r_{i,j}$ is successful if:

1) the SNR at receiver $r_{i,j}$ is no less than the communication threshold γ_c. From (2), this translates to

$$D(t_{i,j}, r_{i,j}) \leqslant \left(\frac{P}{N_0 \gamma_c}\right)^{\frac{1}{\alpha}} =: R_c \quad (4)$$

where R_c is termed as communication radius.

2) the signal from any unintended transmitter $t_{i,k}$ is received at $r_{i,j}$ with an SNR less than a certain threshold γ_i, termed as interference threshold. Equivalently

$$D(t_{i,k}, r_{i,j}) \geqslant \left(\frac{P}{N_0 \gamma_i}\right)^{\frac{1}{\alpha}} =: R_i \quad \forall\ k = 1, \ldots, M_i,\ k \neq j \quad (5)$$

where R_i is termed as interference radius. Note that $0 < \gamma_i < \gamma_c$, thus $R_i > R_c$.

The physical model of our system is denoted by $\Phi(N, (\mathbf{r}_1, \ldots, \mathbf{r}_N), P, \gamma_c, \gamma_i, \alpha, N_0)$. A schedule $\Psi(\cdot)$ is *feasible* if it satisfies the following:

1) Operational constraint: During a time slot, a node can transmit to exactly one node, receive from exactly one node or remain idle.

$$\{t_{i,j}, r_{i,j}\} \cap \{t_{i,k}, r_{i,k}\} = \phi \ \forall\ i = 1, \ldots, C \quad \forall\ 1 \leqslant j < k \leqslant M_i \quad (6)$$

2) Range constraint: Every receiver is within the communication radius of its intended transmitter.

$$D(t_{i,j}, r_{i,j}) \leqslant R_c \ \forall i = 1, \ldots, C \ \forall j = 1, \ldots, M_i \quad (7)$$

A schedule $\Psi(\cdot)$ is *exhaustive* if it satisfies the following:

$$D(j,k) \leqslant R_c \Rightarrow j \to k \in \bigcup_{i=1}^{C} \mathcal{S}_i \text{ and } k \to j \in \bigcup_{i=1}^{C} \mathcal{S}_i \quad \forall\ 1 \leqslant j < k \leqslant N \quad (8)$$

A schedule $\Psi(\cdot)$ is *conflict-free*, if the SINR at every intended receiver exceeds the communication threshold.

$$\text{SINR}_{r_{i,j}} \geqslant \gamma_c \ \forall\ i = 1, \ldots, C,\ \forall\ j = 1, \ldots, M_i \quad (9)$$

B. Graph-Based Scheduling

Link schedules are typically designed from a graph model of the network [4]. The STDMA network $\Phi(\cdot)$ is modeled by a directed graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of vertices and $\mathcal{E}$ is the set of edges. Let $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, where vertex v_j represents the j^{th} node in $\Phi(\cdot)$. In general, $\mathcal{E} = \mathcal{E}_c \cup \mathcal{E}_i$, where $\mathcal{E}_c$ and $\mathcal{E}_i$ denote the set of communication and interference edges respectively. If node k is within node j's communication radius, then there is a communication edge from v_j to v_k, denoted by $v_j \xrightarrow{c} v_k$. If node k is outside node j's communication radius but within its interference radius, then there is an interference edge from v_j to v_k, denoted by $v_j \xrightarrow{i} v_k$. Thus, the mapping from $\Phi(\cdot)$ to $\mathcal{G}(\cdot)$ can be described as follows:

$$D(j,k) \leqslant R_c \Rightarrow v_j \xrightarrow{c} v_k \in \mathcal{E}_c \text{ and } v_k \xrightarrow{c} v_j \in \mathcal{E}_c$$
$$R_c < D(j,k) \leqslant R_i \Rightarrow v_j \xrightarrow{i} v_k \in \mathcal{E}_i \text{ and } v_k \xrightarrow{i} v_j \in \mathcal{E}_i$$

A communication or an interference edge from v_j to v_k will be denoted by $v_j \to v_k$. The subgraph $\mathcal{G}_c(\mathcal{V}, \mathcal{E}_c)$ consisting of communication edges only is termed as the *communication graph* [5].

The schedule $\Psi(\cdot)$ is then designed from the graph $\mathcal{G}(\cdot)$. Specifically, an STDMA link scheduling algorithm is equivalent to assigning a unique color to every communication edge in the graph, such that source-destination pairs corresponding to communication edges with the same color transmit simultaneously in a particular time slot. The traditional method for link assignment requires that two communication edges $v_i \xrightarrow{c} v_j$ and $v_k \xrightarrow{c} v_l$ can be colored the same iff:

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i) vertices v_i, v_j, v_k, v_l are all mutually distinct, i.e., there is no *primary edge conflict,* and

ii) $v_i \rightarrow v_l \notin \mathcal{G}(\cdot)$ and $v_k \rightarrow v_j \notin \mathcal{G}(\cdot)$, i.e, there is no *secondary edge conflict*.

A Graph-Based Scheduling Algorithm (GSA) utilizes various graph coloring methodologies to obtain a schedule devoid of primary and secondary edge conflicts. To maximize the throughput of an STDMA network, a GSA seeks to minimize the total number of colors used to color all the communication edges of $\mathcal{G}(\cdot)$.

C. Limitations of Graph-Based Scheduling

Criteria i) and ii) are not sufficient to guarantee that the resulting schedule $\Psi(\cdot)$ is conflict-free. Importantly, GSAs do not maximize the throughput of an STDMA network because:

1) GSAs can lead to high cumulative interference at a receiver [4], due to hard-thresholding based on R_c and R_i: the SINR at a receiver $r_{i,j}$ decreases with an increase in the number of concurrent transmissions M_i, while R_c and R_i have been defined for a single transmission only. For example, consider Figure 1 with six labeled

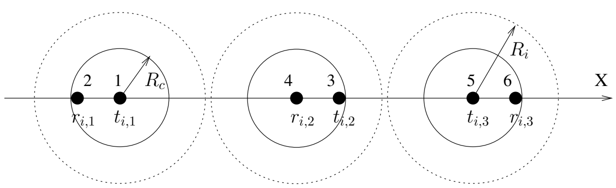

Fig. 1. Graph-Based algorithms can lead to high cumulative interference.

nodes whose coordinates (in meters) are $1 \equiv (-360, 0)$, $2 \equiv (-450, 0)$, $3 \equiv (90, 0)$, $4 \equiv (0, 0)$, $5 \equiv (360, 0)$ and $6 \equiv (450, 0)$. The system parameters are $P = 10$ mW, $\alpha = 4$, $N_0 = -90$ dBm, $\gamma_c = 20$ dB and $\gamma_i = 10$ dB, which yields $R_c = 100$ m and $R_i = 177.8$ m. A GSA will schedule the transmissions $1 \rightarrow 2$, $3 \rightarrow 4$ and $5 \rightarrow 6$ in the same time slot, say the i^{th} time slot, since the resulting graph coloring is devoid of primary and secondary edge conflicts. However, the SINRs at receivers $r_{i,1}$, $r_{i,2}$ and $r_{i,3}$ are 21.26 dB, 18.42 dB and 19.74 dB respectively. From the physical interference model, $t_{i,1} \rightarrow r_{i,1}$ is successful, while $t_{i,2} \rightarrow r_{i,2}$ and $t_{i,3} \rightarrow r_{i,3}$ are unsuccessful. This leads to low throughput.

2) On the other hand, GSAs can be extremely conservative and result in a higher number of colors. For example, with the same system parameters as in 1), consider Figure 2 with four labeled nodes whose coordinates (in meters) are $1 \equiv (0, 0)$, $2 \equiv (50, 0)$, $3 \equiv (220, 0)$ and $4 \equiv (170, 0)$. Consider two transmission requests: $1 \rightarrow 2$ and $3 \rightarrow 4$. If both transmissions are scheduled in the same slot, say the i^{th} time slot, the SINRs at receivers $r_{i,1}$ and $r_{i,2}$ are both equal to 20.91 dB. From the physical interference model, both $t_{i,1} \rightarrow r_{i,1}$ and $t_{i,2} \rightarrow r_{i,2}$ are successful, since signals levels are so

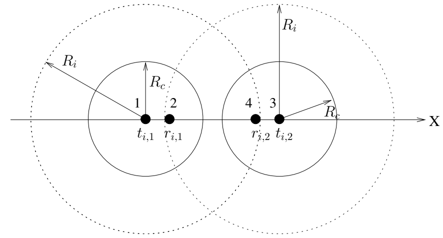

Fig. 2. Graph-Based algorithms can lead to higher number of colors.

high at the receivers that strong interferences can be tolerated. However, due to secondary edge conflicts, a GSA will schedule the above transmissions in different slots, thus decreasing the throughput.

D. Problem Formulation

The communication graph $\mathcal{G}_c(\mathcal{V}, \mathcal{E}_c)$ is an approximation of the STDMA network $\Phi(\cdot)$, while the two-tier graph $\mathcal{G}(\mathcal{V}, \mathcal{E}_c \cup \mathcal{E}_i)$ is a better approximation of $\Phi(\cdot)$. From $\Phi(\cdot)$ and $\mathcal{G}_c(\cdot)$, one can exhaustively determine the optimal STDMA schedule which yields the highest throughput according to the physical interference model. However, this is a combinatorial optimization problem of prohibitive complexity $(O(|\mathcal{E}_c|^{|\mathcal{E}_c|}))$.

To overcome these problems, we propose a new suboptimal algorithm for STDMA link scheduling based on a realistic physical interference model. Our algorithm is based on the communication graph model $\mathcal{G}_c(\mathcal{V}, \mathcal{E}_c)$ as well as SINR computations.

To evaluate the performance of our algorithm and compare it with existing suboptimal STDMA link scheduling algorithms, we define the following metric: spatial reuse. Under the physical interference model, the transmission $t_{i,j} \rightarrow r_{i,j}$ is successful iff (3) is satisfied. The *spatial reuse* of the link schedule $\Psi(\cdot)$ is defined as the average number of successfully received packets per time slot in the STDMA schedule. Thus

$$\text{Spatial Reuse} = \sigma = \frac{\sum_{i=1}^{C} \sum_{j=1}^{M_i} I(\text{SINR}_{r_{i,j}} \geqslant \gamma_c)}{C} \quad (10)$$

where $I(A)$ denote the indicator function for event A, i.e., $I(A) = 1$ if event A occurs, $I(A) = 0$ if event A does not occur.

The essence of STDMA is to have a reasonably large number of concurrent and successful transmissions. For an STDMA network which is operational for a long period of time, say L slots, the total number of successfully received packets is $L\sigma$. Thus, a high value of spatial reuse directly translates to higher network throughput and the number of colors C is relatively unimportant. Hence, spatial reuse[1] turns out to be a crucial metric for the comparison of different STDMA algorithms.

We seek low complexity conflict-free STDMA link scheduling algorithms with high spatial reuse. We only consider

[1]Note that spatial reuse in our system model is analogous to spectral efficiency in digital communication systems.

STDMA schedules which are feasible and exhaustive. Thus, our schedules satisfy (6), (7), (8) and (9).

III. SINR-Based Link Scheduling Algorithm

Motivated by techniques from matroid theory [12], we develop a computationally feasible algorithm with demonstrably high spatial reuse. The essence of our algorithm is to partition the set of communication edges into subsets (forests) and color the edges in each subset sequentially. Our proposed SINR-based link scheduling algorithm is ConflictFreeLinkSchedule (CFLS), which considers the communication graph $\mathcal{G}_c(\mathcal{V}, \mathcal{E}_c)$ and is described in Algorithm 1.

Algorithm 1 ConflictFreeLinkSchedule (CFLS)

```
1: input: Physical network Φ(·), communication graph G_c(·)
2: output: A coloring C : E_c → {1, 2, ...}
3: label the vertices of G_c randomly
4: use successive breadth first searches to partition G_c into
   oriented graphs T_i, 1 ⩽ i ⩽ k
5: for i ← 1 to k do
6:   for j ← 1 to n do
7:     if T_i is out-oriented then
8:       let x = (s, d) be such that L(d) = j
9:     else
10:      let x = (s, d) be such that L(s) = j
11:    end if
12:    C(x) ← FirstConflictFreeColor(x)
13:  end for
14: end for
```

Algorithm 2 integer FirstConflictFreeColor(x)

```
input: Physical network Φ(·), communication graph G_c(·)
output: A conflict-free color
C ← set of existing colors
C_c ← {C(h) : h ∈ E_c, h is colored, x and h have a
  primary edge conflict}
C_cf = C \ C_c
for i ← 1 to |C_cf| do
  r ← i^th color in C_cf
  E_i ← {h : h ∈ E_c, C(h) = r}
  C(x) ← r
  if SINR at all receivers of E_i ∪ {x} exceed γ_c then
    return r
  end if
end for
return |C| + 1
```

In Phase 1 (Line 3), we label all the vertices randomly. Specifically, if $\mathcal{G}_c(\cdot)$ has v vertices, we perform a random permutation of the sequence $(1, 2, \ldots, v)$ and assign these labels to vertices with indices $1, 2, \ldots, v$ respectively.

In Phase 2 (Line 4), the communication graph $\mathcal{G}_c(\cdot)$ is decomposed into what are called as out-oriented and in-oriented graphs $T_1, T_2, \ldots, T_k$ [2]. Each T_i is a forest and every edge of $\mathcal{G}_c(\cdot)$ is in exactly one of the T_i's. This decomposition is achieved by partitioning graph $G_c(\cdot)$, the undirected equivalent of $\mathcal{G}_c(\cdot)$, into undirected forests. The number of forests can be minimized by using techniques from Matroid theory (k-forest problem, [13]). However, this optimal decomposition requires extensive computation. Hence, we adopt the faster albeit non-optimal approach of using successive breadth first searches to decompose $G_c(\cdot)$ into undirected forests. Each undirected forest is further mapped to two directed forests. In one forest, the edges in every connected component point away from the root and every vertex has at most one incoming edge, thus producing an out-oriented graph. In the other forest, the edges in every connected component point toward the root and every vertex has at most one outgoing edge, thus producing an in-oriented graph.

In Phase 3 (Lines 5-14), the oriented graphs are considered sequentially. For each oriented graph, vertices are considered in increasing order by label and the unique edge associated with each vertex is colored using the FirstConflictFreeColor (FCFC) function.

The FCFC function is explained in Algorithm 2. For the edge under consideration x, it discards any color that has an edge with a primary conflict with x. Among the residual set of colors, we choose the first color such that the resulting SINRs at the receiver of x and the receivers of all co-colored edges exceed the communication threshold γ_c. If no such color is found, we assign a new color to x. Thus, this function guarantees that the ensuing schedule is conflict-free.

IV. Performance Results

A. Simulation Model

In our simulation experiments, every node location is generated randomly, using a uniform distribution for its X and Y coordinates in the deployment area. For a fair comparison of our algorithm with Truncated Graph-Based Scheduling Algorithm (TGSA) [4], we assume that the deployment region is a circular region of radius R. Thus, if (X_j, Y_j) are the Cartesian coordinates of the j^{th} node, then $X_j \sim U[-R, R]$ and $Y_j \sim U[-R, R]$ subject to $X_j^2 + Y_j^2 \leqslant R^2$. After generating random positions for N nodes, we have complete information of $\Phi(\cdot)$. Using (4) and (5), we compute R_c and R_i, and thus map the network $\Phi(\cdot)$ to the graph $\mathcal{G}(\cdot)$. Once the schedule $\Psi(\cdot)$ is computed by every algorithm, σ is computed using (10). System parameters are chosen based on their prototypical values in wireless networks [8]. For a given set of system parameters, we calculate the spatial reuse by averaging σ over 1000 randomly generated networks. Keeping all other parameters fixed, we observe the effect of increasing the number of nodes on the spatial reuse. In our experiments, we compare the performance of the following algorithms: ArboricalLinkSchedule [2] (ALS), TGSA [4] and CFLS.

B. Performance Comparison under Path Loss Model

We assume that $R = 500$ m, $P = 10$ mW, $\alpha = 4$, $N_0 = -90$ dBm, $\gamma_c = 20$ dB and $\gamma_i = 10$ dB. Thus, $R_c = 100$ m and $R_i = 177.8$ m. We vary the number of nodes from 30 to 110 in steps of 5. Figure 3 plots the spatial reuse vs. number

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of nodes for all the algorithms.

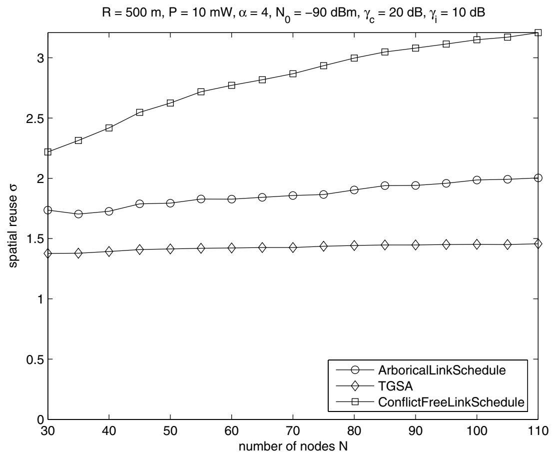

Fig. 3. Spatial reuse vs. number of nodes.

For the ALS algorithm, we observe that spatial reuse increases very slowly with increasing number of nodes.

The spatial reuse of TGSA is 18-27% lower than that of ALS, which we explain as follows. The basis for TGSA is the computation of M^*, the optimal number of transmissions in every slot [4]. M^* is determined by maximizing a lower bound on the expected number of successful transmissions in a time slot. Since the partitioning of a maximal independent set of communication arcs into subsets of cardinality at most M^* is arbitrary and not geography-based, there could be scenarios where the transmissions scheduled in a subset are in the vicinity of each other, resulting in high interference. Also, TGSA yields higher number of colors compared to ALS.

For CFLS, we observe that spatial reuse increases steadily with increasing number of nodes and is consistently 30-60% higher than the spatial reuse of ALS and TGSA.

C. Performance Comparison under Realistic Conditions

In a realistic wireless environment, channel impairments like multipath fading and shadowing affect the received SINR at a receiver. In this section, we compare the performance of the ALS, TGSA and CFLS algorithms in a wireless channel which experiences Rayleigh fading and lognormal shadowing.

In the absence of fading and shadowing, the SINR at receiver $r_{i,j}$ is given by (1). We assume that every algorithm (ALS, TGSA and CFLS) considers only path loss in the channel prior to constructing the graph $\mathcal{G}(\cdot)$ and computing the link schedule. However, when evaluating the performance of each algorithm, we take into account the fading and shadowing channel gains between every pair of nodes. Specifically, for computing the spatial reuse using (10), we assume that the (actual) SINR at receiver $r_{i,j}$ is given by

$$\text{SINR}_{r_{i,j}} = \frac{\frac{P}{D^\alpha(t_{i,j},r_{i,j})} V(t_{i,j},r_{i,j}) 10^{W(t_{i,j},r_{i,j})}}{N_0 + \sum_{\substack{k=1 \\ k\neq j}}^{M_i} \frac{P}{D^\alpha(t_{i,k},r_{i,j})} V(t_{i,k},r_{i,j}) 10^{W(t_{i,k},r_{i,j})}}$$

where random variables $V(\cdot)$ and $W(\cdot)$ correspond to channel gains due to Rayleigh fading and lognormal shadowing respectively. We assume that $\{V(k,l)|1 \leqslant k,l \leqslant N, k \neq l\}$ are independent and identically distributed (i.i.d.) random variables with probability density function (pdf) $f_V(v) = \frac{1}{\sigma_V^2} e^{\frac{-v}{\sigma_V^2}} u(v)$ [14], where $u(\cdot)$ is the unit step function. Also, $\{W(k,l)|1 \leqslant k,l \leqslant N, k \neq l\}$ are assumed to be i.i.d. zero mean Gaussian random variables with pdf $f_W(w) = \frac{1}{\sqrt{2\pi}\sigma_W} e^{\frac{-w^2}{2\sigma_W^2}}$ [15]. $V(\cdot)$ and $W(\cdot)$ are independent of each other and also independent of the node locations.

Our simulation model and system parameters are exactly as described in Sections IV-A and IV-B. In addition, we assume $\sigma_V^2 = \sigma_W^2 = 1$. Figure 4 plots the spatial reuse vs. number of nodes for all the algorithms.

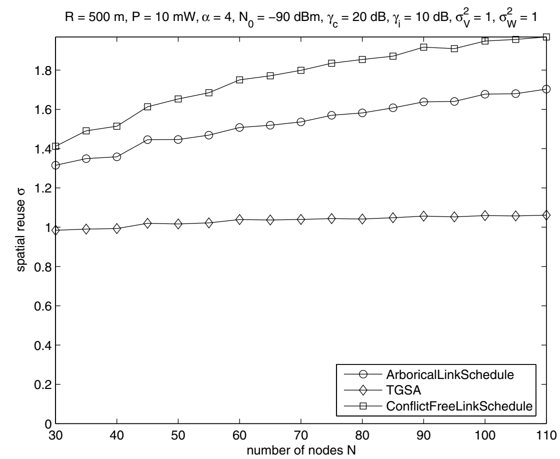

Fig. 4. Spatial reuse vs. number of nodes under multipath fading and shadowing channel conditions.

From Figures 3 and 4, we observe that spatial reuse decreases by 20-40% in a channel experiencing multipath fading and shadowing effects, which we explain as follows. Since the channel gains between every pair of nodes are independent of each other, it is reasonable to assume that the interference power at a typical receiver remains almost the same as in the non-fading case: even if the power received from few transmitters is low, the power received from other transmitters will be high; thus the interference power remains constant. Consequently, the change in SINR is determined by the change in received signal power only. If the received signal power is higher compared to the non-fading case, the transmission is anyway successful and spatial reuse remains unchanged (see (10)). However, if the received signal power is lower, the transmission is now unsuccessful and spatial reuse decreases. Hence, on average, the spatial reuse decreases.

Finally, from Figure 4, we observe that CFLS achieves 8-18% higher spatial reuse than ALS and 40-85% higher spatial reuse than TGSA, under realistic channel conditions.

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V. Analytical Results

In this section, we derive upper bounds on the running time (computational) complexity of the CFLS algorithm. With respect to the communication graph $\mathcal{G}_c(\mathcal{V}, \mathcal{E}_c)$, let:

$$
\begin{aligned}
e &= \text{number of communication edges} \\
v &= \text{number of vertices} \\
\theta &= \text{thickness of the graph} \\
&:= \text{minimum number of planar graphs into which the undirected equivalent of } \mathcal{G}_c(\cdot) \text{ can be partitioned}
\end{aligned}
$$

Lemma 1: An oriented graph T can be colored using no more than $O(v)$ colors using CFLS.

Proof: Since an oriented graph with v vertices has at most v edges, the edges of T can be colored with at most v colors. ■

Lemma 2: For an oriented graph T, the running time of CFLS is $O(v^2)$.

Proof: Assuming that an element can be chosen randomly and uniformly from a finite set in unit time ([16], Chapter 1), the running time of Phase 1 can be shown to be $O(v)$. Since there is only one oriented graph, Phase 2 runs in time $O(v)$. In Phase 3, the unique edge associated with the vertex under consideration is assigned a color using FCFC. From Lemma 1, the size of the set of colors to be examined $|\mathcal{C}_c \cup \mathcal{C}_{cf}|$ is $O(v)$. In FCFC, the SINR is checked only once for every colored edge in the set $\bigcup_{i=1}^{|\mathcal{C}_{cf}|} E_i$ and at most v times for the edge under consideration x. With a careful implementation, FCFC runs in time $O(v)$. So, the running time of Phase 3 is $O(v^2)$. Thus, the total running time is $O(v^2)$. ■

Theorem 1: For an arbitrary graph $\mathcal{G}_c(\cdot)$, the running time of CFLS is $O(ev \log v + ev\theta)$.

Proof: The running time of Phase 1 can be shown to be $O(v)$ [16]. For Phase 2, the optimal partitioning technique of [13] based on Matroids can be used to partition the communication graph $\mathcal{G}_c(\cdot)$ into at most 6θ oriented graphs in time $O(ev \log v)$. Thus, $k \leqslant 6\theta$ holds for Phase 3. From Lemma 2, the first oriented graph T_1 can be colored in time $O(v^2)$. However, consider the coloring of the j^{th} oriented graph T_j, where $2 \leqslant j \leqslant k$. When coloring edge x from T_j using FCFC, conflicts can occur not only with the colored edges of T_j, but also with the edges of the previously colored oriented graphs $T_1, T_2, \ldots, T_{j-1}$ (see [17], Appendix I). Hence, the worst-case size of the set of colors to be examined $|\mathcal{C}_c \cup \mathcal{C}_{cf}|$ is $O(e)$. In FCFC, the SINR is checked only once for every colored edge in the set $\bigcup_{i=1}^{|\mathcal{C}_{cf}|} E_i$ and at most e times for the edge under consideration x. With a careful implementation, FCFC runs in time $O(e)$. Hence, any subsequent oriented graph T_j can be colored in time $O(ev)$. Thus, the running time of Phase 3 is $O(ev\theta)$. Therefore, the overall running time of CFLS is $O(ev \log v + ev\theta)$. ■

VI. Conclusion

In this paper, we have considered an STDMA wireless ad hoc network and presented an algorithm for link scheduling under the physical interference model, with an aim to maximize network throughput. Previous works on STDMA link scheduling algorithms, both graph-based and SINR-based, have only considered schedule length as their performance metric and developed computationally efficient algorithms that minimize the schedule length. Since there is no known relation between schedule length and network throughput, algorithms that minimize the schedule length do not necessarily maximize the network throughput. In contrast to these approaches, we have argued that from a perspective of maximizing network throughput, spatial reuse is an important performance metric for STDMA link scheduling. Moreover, existing works on SINR-based scheduling only consider wireless ad hoc networks using link-layer reliability protocols or wireless ad hoc networks with unconstrained transmission power. The algorithm proposed by us is applicable to a general STDMA wireless ad hoc network with uniform transmission power and results in 40% higher spatial reuse than existing link scheduling algorithms, without compromising on computational complexity. Our approach has the potential to scale with the number of nodes in an STDMA wireless ad hoc network.

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